Plurisubharmonic functions that are piecewise pluriharmonic

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Abstract: Given a finite number of pluriharmonic functions $H_i,\ i = 1, ...k$ in n complex variables, one may form their maximum and obtain a plurisubharmonic function Ψ. It will be piecewise pluriharmonic in the sense that

$$\Psi = \sum_{i=1}^{k} H_i \chi_i,$$

where χ_i is the characteristic function of the open set where $\Psi = H_i$.

I will give a local criterion on a plurisubharmonic function to be such a maximum, or more general to be piecewise plurisubharmonic. This criterion is adapted to a situation where only indirect information is known about Ψ. An example (taken from Hans Rullgrd): suppose that $n = 1$ and $p(z)$ is a generic polynomial with zeroes α_i. If $p(\frac{d\Psi}{dz}) = 0$ then locally near a point p, one has $\Psi(z) = Max\{2\text{Re}\alpha_i z, i \in I\}$ for some set $I \subset [1, ..., n]$ (which depends on p).

I will also sketch some applications to asymptotic behaviour of zeroes of hypergeometric polynomials, in one variable and similar conjectures for several variables.

This builds on joint work with Jan-Erik Bjrk, Julius Borcea, and Boris Shapiro.