HDL and local equilibrium for AZRP with site disorder

C.Bahadoran
Joint with
T. Mountford, K. Ravishankar, E. Saada

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Single queue

- State $\eta \in \mathbb{N}$ number of clients
- Jumps $g \uparrow$, $g(0) = 0 < g(1)$

$$\begin{array}{lll} \eta \to \eta + 1 & \text{rate} & \lambda \\ \eta \to \eta - 1 & \text{rate} & g(\eta) \end{array}$$

- Invariant measure: $\lambda < g(\infty)$,

$$\theta_\lambda(n) = Z(\lambda)^{-1} \frac{\lambda^n}{\prod_{k=1}^n g(k)} = \theta^\rho(n)$$

- Outgoing flux vs. mean density

$$\begin{aligned} \theta_\lambda[\eta] &= Z(\lambda)^{-1} \sum_{n=0}^{+\infty} n\theta_\lambda(n) =: R(\lambda) = \rho \\ \theta_\lambda[g(\eta)] &= Z(\lambda)^{-1} \sum_{n=0}^{+\infty} g(n)\theta_\lambda(n) = \lambda = R^{-1}(\rho) \end{aligned}$$

1. M/M/1
   - $g(n) = n \wedge 1$
   - $R(\lambda) = \dfrac{\lambda}{1-\lambda}$
   - $R^{-1}(\rho) = \dfrac{\rho}{1+\rho}$
2. M/M/∞ (independent clients)
   - $g(n) = n$
   - $R(\lambda) = \lambda$
   - $R^{-1}(\rho) = \rho$

Zero-range process with site disorder
Jackson network with server disorder

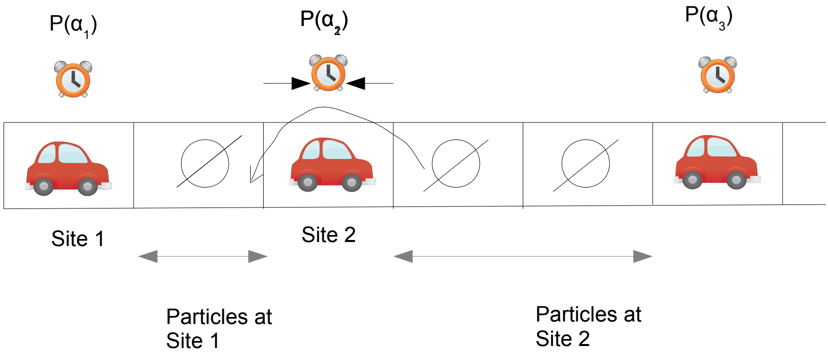
- Configuration $\eta = \{\eta(x),\, x \in \mathbb{Z}\}$
- $\eta(x) \in \mathbb{N}$ nb. particles (clients) at site (server) $x \in \mathbb{Z}$
- $p \in (1/2, 1]$ asymmetry parameter, $q = 1 - p$
- Ergodic environment α with law $Q(d\alpha)$

$$\alpha = \{ \underbrace{\alpha(x)}_{\text{server } x \text{ speed}},\, x \in \mathbb{Z}\} \in (c, 1]^{\mathbb{Z}}, \quad c > 0$$

- Jumps

$$\eta \to \eta - \delta_x + \delta_{x+1} \quad \text{rate} \quad p\alpha(x)\, g[\eta(x)]$$
$$\eta \to \eta - \delta_x + \delta_{x-1} \quad \text{rate} \quad q\alpha(x)\, g[\eta(x)]$$

d-ASEP with particle disorder



Invariant measures

- Product measure with marginal $\theta_{\lambda/\alpha(x)}$ at site x

  $\infty > \lambda \leq c\,g(\infty)$ mean outgoing flow

- Mean flux vs. mean density

$$\overline{R}(\lambda) := \int R\left(\frac{\lambda}{\alpha(0)}\right) dQ(\alpha)$$

$$f(\rho) := \underbrace{(p-q)}_{\text{drift}} R^{-1}(\rho)$$

- Critical density

$$\rho_c := \lim_{\lambda \uparrow c\,g(\infty)} \overline{R}(\lambda) \in [0, +\infty]$$

The current-density function

- Current through origin starting from η

$$\Gamma_0^\alpha(t,\eta) := nb.jumps\, 0 \to 1 - nb.jumps\, 1 \to 0$$

- η^ρ particle configuration with density ρ:

$$\lim_{n\to+\infty} \frac{1}{n}\sum_{x=0}^{n} \eta(x) = \lim_{n\to+\infty} \frac{1}{n}\sum_{x=-n}^{0} \eta(x) = \rho$$

- Define current-density function $\rho \mapsto f(\rho)$

$$f(\rho) := \lim_{t\to+\infty} t^{-1}\Gamma_0^\alpha(t,\eta^\rho)$$

provided exists and depends only on ρ

The current-density function

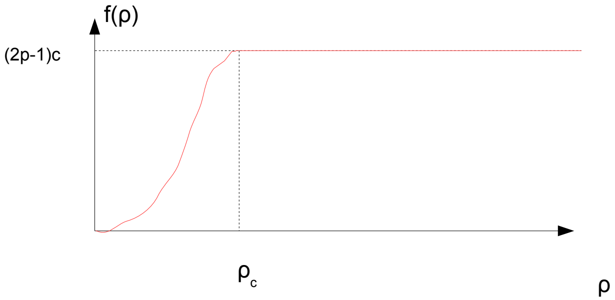
- Thus for $\rho < \rho_c$:

$$f(\rho) = \underbrace{(p - q)}_{\text{drift}} \overline{R}^{-1}(\rho)$$

- What about $\rho \geq \rho_c$?

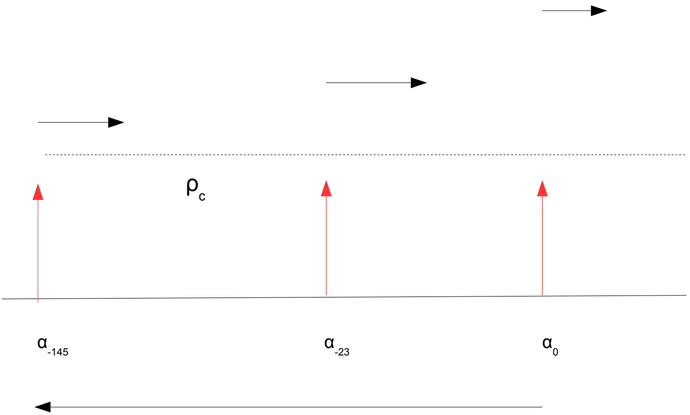
- From now on $g(\infty) = 1$

Flux cutoff



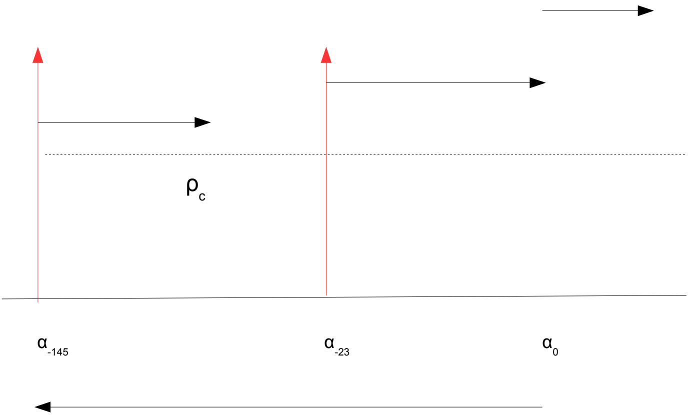
Can show by monotonicity arguments that After ρ_c, current cannot exceed maximum value $(2p-1)c$. What does it mean ?

Heuristics



Successive downward records of α_i
Reaching asymptotic value c at $-\infty$

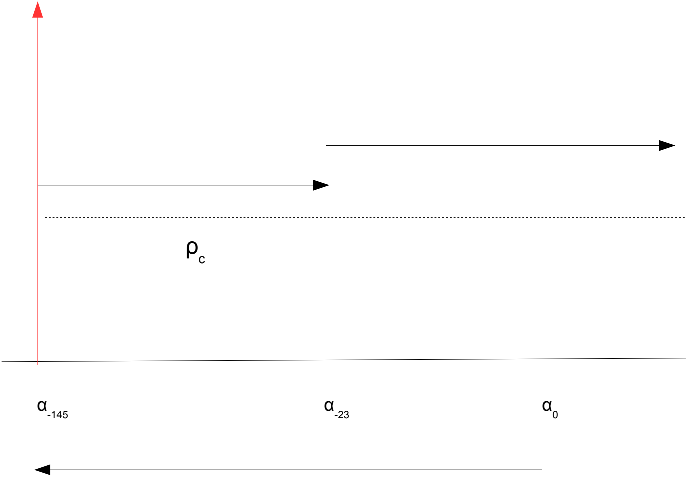
Heuristics



Successive downward records of α_i

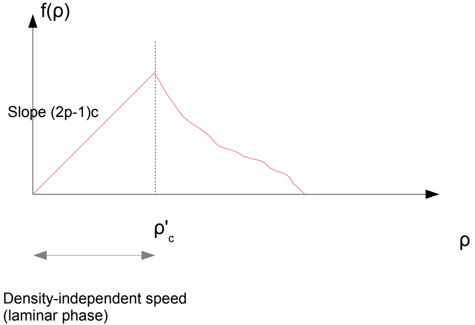
Reaching asymptotic value c at $-\infty$

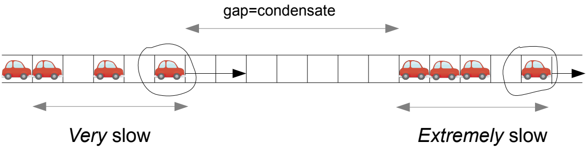
Heuristics



Successive downward records of α_i
Reaching asymptotic value c at $-\infty$

Heuristics (d-ASEP)





Quenched results (I)

Empirical measure, hyperbolic scaling:

$$\pi^N(\eta_{Nt}, dx) := N^{-1} \sum_{y\in\mathbb{Z}} \eta_{Nt}(y)\delta_{y/N}(dx)$$

1. HDL $\pi^N(\eta_0^N, dx) \to \rho_0(.)dx \Rightarrow \pi^N(\eta_{Nt}^N, dx) \stackrel{\mathbf{P}}{\to} \rho(t,.)dx$

$$\partial_t\rho + \partial_x f(\rho) = 0, \quad \rho(0,.) = \rho_0(.) \qquad (1)$$

2. Loc. eq.: If $\rho(t,x) < \rho_c$ cont. point, then $\forall h$ bounded local:

$$\lim_{N\to+\infty} \left\{ \mathbb{E}\left[h(\tau_{\lfloor Nx\rfloor}\eta_{Nt})\right] - \int h(\eta) d\nu^{\tau_{\lfloor Nx\rfloor}\alpha,\rho(t,x)}(\eta) \right\} = 0$$

3. What about $\rho > \rho_c$?

Quenched results (II)

- Supercritical macro point

$$\liminf_{(s,y)\to(t,x)} \rho(s,y) \geq \rho_c$$

- Micro point x_N such that $N^{-1}x_N \to x$
- "Typical": $\forall$ subsequence

$$\tau_{x_N}\alpha \xrightarrow{\text{loc.}} \bar{\alpha}, \quad \text{s.t.} \liminf_{x\to\pm\infty} \bar{\alpha}(x) = c$$

- Conclusion $\forall$ h bounded local:

$$\lim_{N\to+\infty} \left\{ \mathbb{E}\left[h(\tau_{\lfloor Nx\rfloor}\eta_{Nt})\right] - \int h(\eta) d\nu^{\tau_{\lfloor Nx\rfloor}\alpha, \rho_c}(\eta) \right\} = 0$$

Quenched results (III)

Corollary: convergence to invariant measures

Assumption

$$\exists \lim_{n\to+\infty} \frac{1}{n} \sum_{x=-n}^{0} \eta_0(x) = \rho$$

Conclusion

$$\eta_t \stackrel{t\to+\infty}{\longrightarrow} \nu^{\alpha,\rho\wedge\rho_c}$$

Escape of mass [BMRS, to appear in IHP]

Weak convexity assumption on g

THEOREM

1. The limit $\eta_t \to \nu_c$ holds iff.

$$\liminf_{n\to+\infty} \frac{1}{n} \sum_{x=-n}^{0} \eta(x) \geq \rho_c$$

2. Counterexample for non NN RW where $\liminf > \rho_c$ and $\eta_t \not\to \nu_c$ (but does not exceed ν_c)

Now moving frame, no condition on g, but need true $\lim > \rho_c$

Related results

- Convergence of ASEP: Liggett (197?), Andjel (1981), Mountford (2000), B. & Mountford (2006)
- M. Evans (1995), Ferrari & Krug (1996), Krug (2000): disordered ZRP, heuristics
- Benjamini et al. (1996): HDL for disordererd ZRP starting from subcritical local Gibbs.
- Seppälainen (1998) HDL including supercritical data for MM1 and $p = 1$.
- Landim (1993): Conservation of Loc eq. for convex homogeneous ZRP.
- Andjel et al. (2000): escape of mass for MM1 and $p = 1$
- Ferrari & Sisko (2007): upper bound for mass escape in $d \geq 1$

Idea for loc eq.