Renormalized solutions, finite volume and parabolic problem with L^1 data

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Let us consider the following parabolic problem

$$\begin{aligned}
&\frac{\partial u}{\partial t} - \operatorname{div}(\lambda(u)\nabla u) = f \text{ in } Q,\\
&u = 0 \text{ on } \partial\Omega \times (0,T),\\
&u(x,0) = u^0(x)\ \forall x \in \Omega
\end{aligned} \tag{1}$$

with Ω an open bounded (polygonal) subset of $\mathbb{R}^d$, $d \geq 2$, $\boldsymbol{v} \in L^p(\Omega)^d$, T a positive number, $Q = \Omega \times (0,T)$ and $f \in L^1(\Omega)$. Moreover λ is a continuous function such that $\lambda_\infty \geq \lambda(u) \geq \mu > 0$ with λ_∞, μ two positive real numbers.
The main difficulties in dealing with the existence and the uniqueness of a solution to problem (1) are due to the nonlinear character of the operator and to the L^1 data.
For similar problems, the convergence of finite volume schemes is studied in [2]. Adapting the tools developed in the continuous case by Boccardo and Gallouët in [1], the authors proved that the solution of the scheme converges to a solution in the distribution sense.
However it is well known that, in general, such a solution is not unique. The framework of renormalized solutions allows us to have existence, uniqueness and stability results for numerous elliptic or parabolic problems with L^1 data (see [4] for the parabolic case).
In this work, we show that the solution of the scheme converges to the unique renormalized solution of (1). Such a result is already obtained in the elliptic case in [3] where two difficulties have been overcome: prove a discret equivalent to the estimate on the energy and cope with residual terms when passing to the limit. In the parabolic case, new difficulties appear: prove a discrete estimate on the time derivative in order to extract a convergent subsequence using the discrete version of an Aubin's type lemma and pass to the limit in the parabolic term.

References

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