# Mode Coupling in a nonlinear network

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**Abstract**

The graph wave equation arises naturally from conservation laws on a network; there, the usual continuum Laplacian is replaced by the graph Laplacian. This matrix being symmetric and negative yields normal modes that are uncoupled in the linear evolution. When a nonlinearity is present the normal modes will couple. To study this, we consider a wave equation with a cubic defocusing non-linearity on a general network. The model is well-posed. It is close to the $\phi^4$ model in condensed matter physics. Using the normal modes of the graph Laplacian as a basis, we derive amplitude equations and define resonance conditions that relate the graph structure to the dynamics. For cycles and chains, the spectrum of the Laplacian is known; the resonance conditions reduce to a trigonometric identities. Trivial and non trivial solutions of these resonance conditions have been calculated. From these we obtain the amplitude equations coupling the normal modes; these show that the zero mode plays a special role.

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