New approach of Fourier analysis on the Heisenberg group and applications

Abstract :In this joint work with Jean-Yves Chemin and Raphael Danchin, we propose a new approach of the Fourier transform on the Heisenberg group $\mathbb{H}^d$. The basic idea is to take advantage of Hermite functions so as to look at Fourier transform of integrable functions as mappings on the set $\widetilde{\mathbb{H}}^d = \mathbb{N}^d \times \mathbb{N}^d \times \mathbb{R} \setminus \{0\}$ endowed with a suitable distance $\widehat{d}$ (whereas with the standard viewpoint the Fourier transform is a one parameter family of bounded operators on $L^2(\mathbb{R}^d)$). We prove that the Fourier transform of integrable functions is uniformly continuous on $\widetilde{\mathbb{H}}^d$ (for distance $\widehat{d}$), which enables us to extend $\widehat{f}_{\mathbb{H}}$ to the completion $\widehat{\mathbb{H}}^d$ of $\widetilde{\mathbb{H}}^d$, and to get an explicit asymptotic description of the Fourier transform when the 'vertical' frequency tends to 0. We expect our approach to be relevant for adapting to the Heisenberg framework a number of classical results for the $\mathbb{R}^n$ case that are based on Fourier analysis.

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